PARAMETRIC TIME-DOMAIN MEASURES OF LONG-TERM HEART RATE VARIABILITY SIGNAL

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Abstract— **Based on alpha-stable distributional modelling, we have proposed a set of parametric measures to analyze long-term heart rate variability(HRV). The set, in difference from standard measures, accommodates premature beats in the HRV analysis. It preserves the structure of standard measures, redefines many of them and includes new measures. The rationale of the method was demonstrated by simulations. We found that the standard measures SDANN and SDNNIndex were strongly correlated to their parametric counterparts SDARR and SRRIndex, DRRIndex in a group of normal subjects (correlation coefficients 0.979, 0.939, 0.999) and less so in a group of patients (0.942, 0.544, 0.463). Statistical tests indicated that the new parametric measures could better differentiate better between the two groups.**

Keywords— **Heart rate variability, alpha-stable distribution, premature beats.**

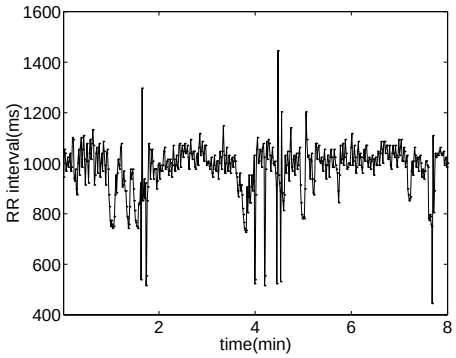

Fig. 1. RR-interval signal with premature ventricular beats.

I. Introduction

Various statistical measures are proposed to analyze long-term (usually 24 h) heart rate variability signal(HRV) or more specifically the RR-interval signal (the sequence of intervals between consecutive R peaks on QRS complexes of the ECG signal). Recommended standard measures [1] are used to analyze long-term HRV signals that consist of normal-to-normal (NN) intervals, i.e. RR intervals that originate exclusively from sinus node depolarizations. In practice, the NN-interval signal is obtained from the RR interval signal from which all 'non-normal' intervals are removed or replaced by interpolated values of neighboring NN samples.

In general, relevant outlier information, such as premature-beats (PB) and arrhythmic episodes, can not be accommodated by the standard analysis as their presence could render most of the measures unreliable. On the other hand removal or replacement of PB from the RR-interval signals would generally distort the correlation structure of the signal especially when the number of PB is high [2].

Development or modification of analysis tools to process the physiological RR-interval (instead of the edited NN-interval signal) would provide an unified view of the HRV. It might contribute amongst others to the study of arrhythmia onset, which often is preceded by PB events. Fig. 1 depicts an RR-interval signal with premature ventricular beats(PVB). A salient characteristic of the signal (Fig. 1) is the presence of impulses.

An apparent advantage of the standard time-domain measures is that they are not explicitly associated to any specific distribution. On the other hand, as in case of impulsive RR-interval signals, the measures could become unreliable. Forcing an αS, or any other, distributional model, carries the risk of model misfit. Nevertheless, the αS model has four distributional parameters with which it could approximate various unimodal distributions. Using the α-stability test [3], it was shown that the alpha-stable (αS) distribution provided a good fit to the HRV data when modelling short-term NN-interval signals [4] and RR-interval signals with PB [5].

Other heavy-tailed distributions, such as generalized Gaussian distribution and t-distribution, could also be used to model the RR-interval signals but they would lack the flexibility of the αS model (being symmetrical and characterized by a single parameter). On the other hand a Gaussian-mixture distribution (e.g. in a signal plus noise model) could be a viable alternative. of PVB and PAB).

Here we applied the αS distributional model to long-term RR-interval signal and derived appropriate measures to analyze it. Some of the measures were found correlated to standard measures whilst the others provided additional information able to statistically differentiate between groups of normal subjects and patients.

II. Method

A. Alpha-stable distributional modelling

Alpha-stable (αS) distribution [6] $S_\alpha(\beta,\gamma,\delta)$ is a family of heavy-tailed distributions that possesses many attractive properties. So, the generalized central limit theorem states that the αS distribution is the only possible limit distribution of the sum of randomly, independently and identically distributed data. The distribution have been successfully applied to model impulsive noise of various sources [7]

The αS distribution is defined by the characteristic function

$$\phi(t) = \exp\{\mathrm{j}\delta\mathrm{t} - \gamma|\mathrm{t}|^\alpha[1 - \mathrm{j}\beta\mathrm{sign}(t)\omega(t,\alpha)] \tag{1a}$$

where

$$\omega(t,\alpha) = \begin{cases} \tan(\frac{\alpha\pi}{2}), & \alpha \neq 1 \\ -\frac{2}{\pi}\log|t|, & \alpha = 1, \text{ and} \end{cases} \tag{1b}$$

$$\mathrm{sign}(t) = \begin{cases} t/|t|, & t \neq 0 \\ 0, & t = 0 \end{cases} \tag{1c}$$

and

$$\alpha \in (0,2],\ \beta \in [-1,1],\ \gamma \in (0,\infty) \text{ and } \delta \in (-\infty,\infty). \tag{1d}$$

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Abstract

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The distribution's parameters are: (i) the *characteristic exponent* α, (ii) the *symmetry parameter* β, (iii) the *dispersion* γ, and (iv) the *location* δ. The αS distribution becomes symmetric for $\beta = 0$. Gaussian($\alpha = 2, \beta = 0$), Cauchy($\alpha = 1, \beta = 0$) and Lévi ($\alpha = 1/2, \beta = 1$) distributions are special cases of the αS distribution. Only (fractional) moments m of order $-1 < m < \alpha$ do exist. The variance ($m = 2$) and mean (for $m < 1$) are infinite.

The impulsive characteristics of the RR-interval signal, with and without PB, could be accommodated by the heavy tails and the skewness of the αS distributions [4], [5]. Thus, the parameter α could depict the *impulsiveness* of the RR interval signal, β the *symmetry* of the data, γ the dispersion of the RR interval signal (the variance that does not exists), and δ the *location* of data.

On the following, instead of dispersion γ, we would use the measures

$$\varsigma = \gamma^{\frac{1}{2}}\sqrt{2} \tag{2}$$

$$c = \gamma^{\frac{1}{\alpha}}\sqrt{2} \tag{3}$$

named respectively *deviation*(D) and *scale*(S). The measure $c/\sqrt{2}$ can be directly obtained from the linear regression of $\log(-\log(|\phi(t)|^2)$ (1a) [8]. In case of Gaussian distribution($\alpha = 2$) they would equal the distribution parameter σ.

B. Standard and nonparametric αS measures

First, we have listed the recommended standard measures [1] using the notations $\hat{\sigma}$ and $\hat{\mu}$ for respectively sample standard deviation(SD) and sample mean.

- SDNN = $\hat{\sigma}$, the SD of all NN intervals.
- RMSSD $\simeq$ SDSD = $\hat{\sigma}(\Delta)$, where Δ is the first backward difference of the NN-interval signal.
- SDANN = $\hat{\sigma}(\hat{\mu}_K)$, where $\hat{\mu}_K$ indicates the array $\{\ \hat{\mu}_1, \hat{\mu}_2, \ldots, \hat{\mu}_K\ \}$ of sample averages of sequential data segments of equal five minutes lengths.
- HRV index = N/H, where N is the number of NN intervals and H the maximum value of the histogram.

Another related measures, to be referred later on, is

- SDNNIndex = $\hat{\mu}(\hat{\sigma}_K)$, where $\hat{\sigma}_K$ indicates the array $\{\hat{\sigma}_1, \hat{\sigma}_2, \ldots, \hat{\sigma}_K\}$ and $\hat{\sigma}_k$ is the SD of data on segment k.

Then, we modified the above time-domain measures based on the assumption that data could be modelled by an αS distribution. Whenever possible we kept the standard notation and simply substituted the notation 'RR' for 'NN'. Otherwise, the notation 'SD' was substituted by the symbols 'D' and 'S' respectively for measures of deviation and scale. The mapping of standard measures (mathematically described inside square brackets) yielded

$$\text{SDNN}[\hat{\sigma}] \longrightarrow \text{DRR}[\hat{\varsigma}] \tag{4}$$

$$\text{SDSD}[\hat{\sigma}(\hat{\Delta})] \longrightarrow \text{DSD}[\hat{\varsigma}(\hat{\Delta})] \tag{5}$$

$$\text{SDANN}[\hat{\sigma}(\hat{\mu}_K)] \longrightarrow \text{SDARR}[\hat{\sigma}(\hat{\delta}_K)] \tag{6}$$

$$\text{SDNNIndex}[\hat{\mu}(\hat{\sigma}_K)] \longrightarrow \text{DRRIndex}[\hat{\mu}(\hat{\varsigma_K})] \tag{7}$$

The SDNN and SDNNIndex measures could alternatively be mapped to

$$\text{SDNN}[\hat{\sigma}] \longrightarrow \text{SRR}[\hat{c}] \tag{8}$$

$$\text{SDNNIndex}[\hat{\mu}(\hat{\sigma}_K)] \longrightarrow \text{SRRIndex}[\hat{\mu}(\hat{c_K})] \tag{9}$$

The set of global measures, including DRR and SRR, becomes

$$\hat{\alpha},\ \hat{\beta},\ \hat{\varsigma}\ \textit{or}\ \hat{c},\ \text{and}\ \hat{\delta}. \tag{10}$$

This set could parameterize the probability density function (PDF) of the data and provide similar information to the geometrical(distributional) measures [1].

Care should be exerted as the set of global measures might not be appropriate when considering 24-h HRV recordings, which are basically non-stationary. One could use instead the measures

$$\hat{\mu}(\hat{\alpha}_K),\ \hat{\mu}(\hat{\beta}_K),\ \text{DRRIndex or SRRIndex, and}\ \hat{\mu}(\hat{\delta}_K) \tag{11}$$

$$\hat{\sigma}(\hat{\alpha}_K),\ \hat{\sigma}(\hat{\beta}_K),\ \hat{\sigma}(\hat{\varsigma}_K)\ \text{or}\ \hat{\sigma}(\hat{c}_K),\ \text{and SDARR} \tag{12}$$

which capture correspondingly the low and high-frequency variations of αS parameters. The sequences $\hat{\alpha}_K$, $\hat{\beta}_K$, $\hat{\varsigma}_K$, and $\hat{c}_K$ denote the corresponding parameter arrays estimated from five minutes segments.

The information contained in the HRV-Index, which was simply renamed to RR-index, could be captured by the parameters of the αS distribution and might not be included on the set of parametric measures.

The obtain the above parametric measures the RR-interval signal would need to be whitened. Assuming that data can be modelled by a linear autoregressive model $A(z)$ we estimated the model parameters using the least-squares method, which was shown to be applicable to symmetrical αS processes in [7], and whitened the signal by the inverse filter $A^{-1}(z)$. The trimmed mean($\pm$ 25%) was preliminarily removed from the correlated signal and was added back to the whitened signal, which was also scaled to have the SD of the original signal.

C. Simulation

We illustrated the differences between distribution-free measures and parametric measures through Monte Carlo simulations. Five hundred data segments of length 1000 samples were generated from a t-distribution with various degrees of freedom(DF). The t-distribution was specifically chosen as it's PDF varies from a heavy tailed distribution to a Gaussian one with the increase of DF; for two DF the t-distribution, likewise the αS distribution, has infinite variance.

We compared the non-parametric SD ($\hat{\sigma}$) with measures D ($\hat{\varsigma}$) and S ($\hat{c}$) of the αS distribution model; the other measures are derived from these basic measures. Thus, SD enters SDNN, SDSD and SDNN-index measures whereas D enters DRR, DSD and DRR-index; similarly S is included in SRR, SSD, and SRR-index.

The simulation likens the clinical situations where the HRV signal could be 'well-behaved' or display impulsive characteristics. One could could either use the non-parametric estimate $\hat{\sigma}$ (after removing all non-normal beats) or, on indication of impulses, model the data by an heavy tailed αS distribution and estimate the parameters ς and c.

D. HRV analysis

We evaluated the proposed method in 18 subjects which underwent 24-hour Holter ECG recording(DelMar Avionics$^{\text{TM}}$), because of perceived palpitations. Eleven subjects were classified normal (group I) and the seven others, which had more than 1000 PVB per 24h, as non-normal (group II).

The recordings begun around 10 AM (except two records which started at 2 PM). The HRV signals were extracted from

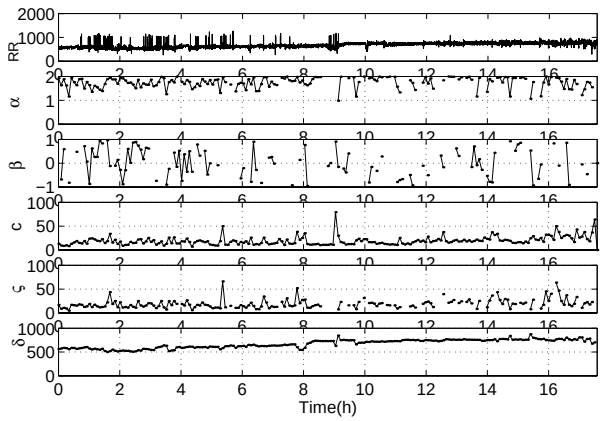

Fig. 2. The RR-interval signal (top) and sequences of αS parameters. evolution

the ECG signals sampled at 128 Hz and quantized to 12 bit resolution. Annotation of beat types was done on the ECG signal by an experienced operator. The length of HRV data files was about 100000 samples. First we rejected the artifacts to obtain the RR-interval signals, and then the non-normal beats to obtain the NN-interval signals. In average, the number of rejected beats in group I was 0.558% (0.478%) for the NN-interval signals and 0.500% (0.468%) for the RR-interval signals. In group II the corresponding numbers were respectively 2.412% (3.189%) and 21.39% (15.176%)—proportions in parenthesis denote the SD in percentage.

From the standard analysis we obtained first the SDNN, RMSSD, and SDNNIndex measures on both NN and, for comparison, RR intervals signals. Then, we analyzed the long-term RR-interval signals from each group using the αS parametric measures obtained from the sequences $\hat{\alpha}_K$, $\hat{\beta}_K$, $\hat{c}_K$, $\hat{\varsigma}_K$, and $\hat{\delta}_K$. Fig. 2 shows these sequences for a subject in group; each dot in the graph corresponds to a parameter value (of a five minutes segment length), whereas its absence indicates that the corresponding parameter is out of range, either due to estimation error or model misfit.

III. Results

Results of simulations (section II-C) are shown in Table I. Table entries are the sample mean, SD(in parenthesis) and coefficient of variation(CV)(in percentage) of the estimates of σ, ς, c and α. The SD of the parametric estimates were lower then the SD of σ (Table I), more so when the number of DF decreases, i.e. the signal gets more impulsive. When the distribution approached the Gaussian distribution (DF $\gtrsim$ 30), the estimates converged to SD, which in this case yielded the best estimate. The coefficients of variations showed a similar behavior. The parameter α had the lowest CV, whereas CV of c and ς were lower than $\hat{\sigma}$ respectively for DF ≤ 7 and DF ≤ 5.

It should be mentioned that neither $\hat{\varsigma}$ nor c were (thought to be) estimates of σ. Except for normal distributed data they do not not equal the SD. The information contained in σ was spread to the parameters ς and α, or mapped to the scale parameter c; in both cases with smaller estimation error variance.

Table II shows the results of αS analysis. The upper part of the table delineates the group I of normal subjects whereas the lower part the group II of patients.

We evaluated first the relations of measures SDNN, SDANN and SDNNIndex from the sets of NN and (unproperly) of RR interval signals. The correlation coefficients (CC) were respectively 0.9994, 1, 0.9996 in group I and 0.9908, 0.4464, 0.9414 in group II.

TABLE I

Estimation of σ and parametric measures of data from an "arbitrary" distribution

DF	σ	$\hat{\sigma}$	$\hat{\varsigma}$	$\hat{c}$	$\hat{\alpha}$
2	∞	3.223	1.270	1.217	1.432
		(2.927)	(0.033)	(0.047)	(0.053)
		90.816%	3.792%	3.868%	3.754%
3	1.732	1.710	1.202	1.162	1.653
		(0.247)	(0.033)	(0.041)	(0.048)
		14.444%	3.573%	3.608%	2.956%
5	1.291	1.292	1.126	1.102	1.823
		(0.053)	(0.029)	(0.033)	(0.038)
		4.102%	3.830%	3.050%	2.106%
7	1.183	1.185	1.093	1.076	1.886
		(0.036)	(0.028)	(0.031)	(0.030)
		3.038%	3.810%	2.951%	1.630%
10	1.118	1.117	1.065	1.054	1.929
		(0.031)	(0.027)	(0.029)	(0.025)
		2.775%	3.866%	2.814%	1.313%
30	1.035	1.036	1.020	1.017	1.980
		(0.024)	(0.025)	(0.026)	(0.017)
		2.316%	4.057%	2.603%	0.892%

Second, the CC of SDANN, SDARR as well as of SDNNIndex, SRRIndex and SDNNIndex, DRRIndex are given in Table III. The strong correlation between standard and αS measures in normal subjects (group I), decreased markedly (SDARR and DRRIndex), in Group II. Fig. 3 shows the SDNNIndex and SRRIndex measures of groups I and II.

Third, the statistical differences between groups were assessed using the Wilcoxon rank sum test (which assesses a median shift between two identical distributions) and a directional parametric t-test. Both tests yielded equivalent results.

From the standard measures SDANN, SDNNIndex, SDNN and the modified measures SDARR, SRRIndex and DRRIndex measures, only the SDNNIndex and SRRIndex were able to reject the null hypothesis (p-values 0.04 and 0.02). The measures in the Group I were smaller then those in Group II.

From the new measures(left part on Table II) we found that $\hat{\mu}(\hat{\alpha}_K)$, was higher in the normal group and $\hat{\sigma}(\hat{\alpha}_K)$ lower. The parameter $\hat{\mu}(\hat{\beta}_K)$could not differentiate the groups whereas $\hat{\sigma}(\hat{\beta}_K)$ was larger in the normal group.

IV. Discussion

The $\hat{\mu}(\hat{\alpha}_K)$ was, as expected, higher in Group I then in Group II, indicating that the HRV in normal group is less impulsive (fewer PB) then in patients' group. The measure $\hat{\sigma}(\hat{\alpha}_K)$ was lower in Group I, which might indicate a smooth long-term characteristic as opposed to a more burst-like characteristic on Group II. The higher value of parameter $\hat{\sigma}(\hat{\beta}_K)$ would need to be examined closer—it might be an intrinsic feature of HRV or might indicate the dominance of PB in β or reflect the estimation error of β when α get closer the value two(Fig. 2).

The estimation of the parametric measures is less straightforward than that of standard measures. The degree of whitening on each data segments might vary, effecting the quality of the αS estimates. Also, scaling of the the whitened data to the SD of the HRV signal is rather artificial and could introduce inaccuracies.

TABLE II

PARAMETRIC α-STABLE ANALYSIS

$\hat{\mu}(\hat{\alpha}_K)$ $(\hat{\sigma}(\hat{\alpha}_K))$	$\hat{\mu}(\hat{\beta}_K)$ $(\hat{\sigma}(\hat{\beta}_K))$	$\hat{\mu}(\hat{c}_K)$ $(\hat{\sigma}(\hat{c}_K))$ SRRIndex	$\hat{\mu}(\hat{\varsigma}_K)$ $(\hat{\sigma}(\hat{\varsigma}_K))$ DRRIndex	$\hat{\mu}(\hat{\delta}_K)$ $(\hat{\sigma}(\hat{\delta}_K))$ (SDARR)
1.739	-0.007	26.782	18.920	673.837
(0.229)	(0.605)	(2.734)	(9.720)	(91.757)
1.780	-0.130	47.021	30.603	801.044
(0.212)	(0.566)	(19.189)	(13.322)	(137.139)
1.844	-0.046	30.292	23.080	705.104
(0.161)	(0.527)	(12.534)	(9.391)	(95.207)
1.791	-0.096	38.824	22.391	763.394
(0.195)	(0.608)	(24.460)	(10.733)	(95.331)
1.823	-0.073	46.838	34.836	822.834
(0.187)	(0.570)	(18.633)	(15.189)	(130.209)
1.798	0.037	67.350	45.881	1033.975
(0.227)	(0.532)	(27.28)	(22.781)	(174.183)
1.779	-0.195	29.849	21.025	839.755
(0.199)	(0.567)	(15.520)	(11.450)	(109.935)
1.817	0.033	41.453	30.674	765.958
(0.220)	(0.562)	(19.137)	(14.364)	(101.651)
1.828	-0.003	42.079	31.585	765.787
(0.199)	(0.557)	(18.783)	(15.528)	(102.264)
1.694	-0.230	70.579	42.436	889.601
(0.299)	(0.496)	(25.710)	(24.722)	(175.205)
1.779	-0.222	19.079	13.963	729.465
(0.246)	(0.444)	(10.134)	(7.489)	(56.162)
1.146	-0.187	94.796	28.797	794.847
(0.598)	(0.440)	(225.219)	(48.342)	(104.284)
0.994	-0.025	293.648	32.089	760.863
(0.544)	(0.390)	(2294.476)	(47.087)	(99.103)
1.508	0.011	64.7412	27.784	796.781
(0.322)	(0.425)	(37.429)	(24.098)	(146.766)
0.848	-0.356	306.467	28.062	901.394
(0.428)	(0.376)	(105.276)	(58.254)	(73.432)
1.718	-0.152	85.831	28.527	702.110
(0.298)	(0.490)	(78.125)	(15.822)	(113.814)
1.391	0.312	45.691	20.147	734.848
(0.255)	(0.342)	(28.411)	(22.357)	(73.201)
1.395	0.242	38.018	19.771	736.372
(0.291)	(0.322)	(25.834)	(23.923)	(104.496)

TABLE III

CORRELATION COEFFICIENTS BETWEEN STANDARD AND PARAMETRIC αS MEASURES

Group	SDNNIndex/ SRRIndex	SDNNIndex/ DRRIndex	SDANN/ SDARR
I	0.9771	0.9391	0.9973
II	0.9421	0.5446	0.4631

Further assessment on larger populations would be needed to fully validate the utility of the αS parametric measures in clinical practice.

In conclusion, this study has introduced a set of parametric measures for the analysis of long-term HRV. The measures were found highly correlated to the standard measures on HRV of normal subjects and provided additional information on HRV of patients with ventricular premature beats.

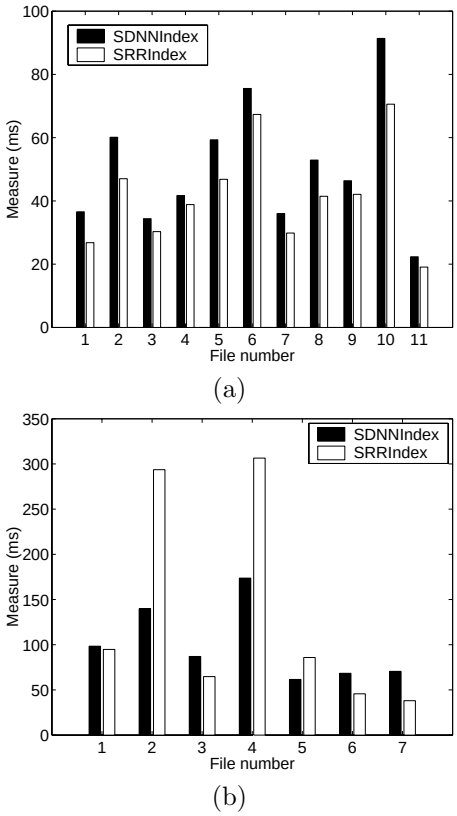

Fig. 3. The SDNNIndex and SRRIndex measures (a)group I and (b) group II.

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